## PyLops-distributed

Oct 09, 2020

Getting started:

1 History 3
Index

Note: This library is under early development.
Expect things to constantly change until version v1.0.0.

This library is an extension of PyLops for distributed operators.
As much as numpy and scipy lie at the core of the parent project PyLops, PyLops-distributed heavily builds on top of Dask, and more specifically Dask arrays.

Doing so, linear operators can be parallelized across several processes on a single node or across multiple nodes. Their forward and adjoint are first lazily built as directed acyclic graphs and evaluated only when requested by the user (or automatically within one of our solvers).
Most of the operators and solvers in PyLops-distributed mirror their equivalents in PyLops and users can seamlessly switch between PyLops and PyLops-distributed or even combine operators acting locally with distributed operators.
Here is a simple example showing how a diagonal operator can be created, applied and inverted using PyLops:

```
import numpy as np
from pylops import Diagonal
n = 10
x = np.ones(n)
d = np.arange(n) + 1
Dop = Diagonal(d)
# y = Dx
y = Dop*x
# x = D'y
xadj = Dop.H*y
# xinv = D^-1 y
xinv = Dop / y
```

and similarly using PyLops-distributed:

```
import numpy as np
import dask.array as da
import pylops_distributed
from pylops_distributed import Diagonal
# set-up client
client = pylops_distributed.utils.backend.dask()
n}=1
x = da.ones(n, chunks=(n//2,))
d = da.from_array(np.arange(n) + 1, chunks=(n//2, n//2))
Dop = Diagonal(d)
# y = Dx
y = Dop*x
# x = D'y
xadj = Dop.H*y
# xinv = D^-1 y
xinv = Dop / y
```

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```
da.compute((y, xadj, xinv))
client.close()
```

It is worth noticing two things at this point:

- In this specific case we did not even need to reimplement the Diagonal operator. Calling numpy operations as methods (e.g., x. sum ()) instead of functions (e.g., np. sum (x)) makes it automatic for our operator to act as a distributed operator when a dask array is provided instead. Unfortunately not all numpy functions are also implemented as methods: in those cases we reimplement the o perator directly within PyLops-distributed.
- Using * and . $\mathrm{H} *$ is still possible also within PyLops-distributed, however when initializing an operator we will need to decide whether we want to simply create dask graph or also evaluation. This gives flexibility as we can decide if and when apply evaluation using the compute method on a dask array of choice.


## CHAPTER 1

History

PyLops-Distributed was initially written and it is currently maintained by Equinor It is an extension of PyLops for large-scale optimization with distributed linear operators that can be tailored to our needs, and as contribution to the free software community.

### 1.1 Installation

You will need Python 3.5 or greater to get started.

### 1.1.1 Dependencies

Our mandatory dependencies are limited to:

- numpy
- scipy
- numba
- dask
- pylops

We advise using the Anaconda Python distribution to ensure that these dependencies are installed via the Conda package manager.

### 1.1.2 Step-by-step installation for users

## Python environment

Stable releases on PyPI and Conda coming soon...
To install the latest source from github:
>> pip install https://git@github.com/equinor/pylops-distributed.git@master
or just clone the repository

```
>> git clone https://github.com/equinor/pylops-distributed.git
```

or download the zip file from the repository (green button in the top right corner of the main github repo page) and install PyLops from terminal using the command:

```
>> make install
```


### 1.1.3 Step-by-step installation for developers

Fork and clone the repository by executing the following in your terminal:

```
>> git clone https://github.com/your_name_here/pylops-distributed.git
```

The first time you clone the repository run the following command:

```
>> make dev-install
```

If you prefer to build a new Conda enviroment just for PyLops, run the following command:

```
>> make dev-install_conda
```

To ensure that everything has been setup correctly, run tests:

```
>> make tests
```

Make sure no tests fail, this guarantees that the installation has been successfull.
If using Conda environment, always remember to activate the conda environment every time you open a new bash shell by typing:

```
>> source activate pylops-distributed
```


### 1.2 Tutorials

### 1.2.1 Marchenko redatuming by inversion

This example shows how to set-up and run the pylops_distributed.waveeqprocessing. Marchenko inversion using synthetic data.

Data are first converted to frequency domain and stored in the high-performance format Zarr. This allows lazy loading using a Dask array and distributing over frequencies the computation of the various Fredholm integrals involved in the forward model.

```
# sphinx_gallery_thumbnail_number = 3
import warnings
import numpy as np
import dask.array as da
import matplotlib.pyplot as plt
```

```
from scipy.signal import convolve
from pylops_distributed.waveeqprocessing import Marchenko
warnings.filterwarnings('ignore')
plt.close('all')
```

Let's start by defining some input parameters and loading the test data

```
# Input parameters
inputfile = '../testdata/marchenko/input.npz'
inputzarr = '../testdata/marchenko/input.zarr'
vel = 2400.0 # velocity
toff = 0.045 # direct arrival time shift
nsmooth = 10 # time window smoothing
nfmax = 1000 # max frequency for MDC (#samples)
niter = 10 # iterations
inputdata = np.load(inputfile)
# Receivers
r = inputdata['r']
nr = r.shape [1]
dr = r[0, 1]-r[0, 0]
# Sources
s = inputdata['s']
ns = s.shape[1]
ds = s[0, 1]-s[0, 0]
# Virtual points
vs = inputdata['vs']
# Density model
rho = inputdata['rho']
z, x = inputdata['z'], inputdata['x']
# Reflection response in frequency domain (R[f, s, r])
R_fft = da.from_zarr(inputzarr)
print('R_fft:', R_fft)
# Subsurface fields
Gsub = inputdata['Gsub']
G0sub = inputdata['G0sub']
wav = inputdata['wav']
wav_c = np.argmax(wav)
t = inputdata['t']
ot, dt, nt = t[0], t[1]-t[0], len(t)
Gsub = np.apply_along_axis(convolve, 0, Gsub, wav, mode='full')
Gsub = Gsub[wav_c:][:nt]
G0sub = np.apply_along_axis(convolve, 0, G0sub, wav, mode='full')
G0sub = G0sub[wav_c:] [:nt]
plt.figure(figsize=(10, 5))
```

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```
plt.imshow(rho, cmap='gray', extent=(x[0], x[-1], z[-1], z[0]))
plt.scatter(s[0, 5::10], s[1, 5::10], marker='*', s=150, c='r', edgecolors='k')
plt.scatter(r[0, ::10], r[1, ::10], marker='v', s=150, c='b', edgecolors='k')
plt.scatter(vs[0], vs[1], marker='.', s=250, c='m', edgecolors='k')
plt.axis('tight')
plt.xlabel('x [m]')
plt.ylabel('y [m]')
plt.title('Model and Geometry')
plt.xlim(x[0], x[-1])
fig, axs = plt.subplots(1, 2, sharey=True, figsize=(12, 9))
axs[0].imshow(Gsub, cmap='gray', vmin=-1e6, vmax=1e6,
    extent=(r[0, 0], r[0, -1], t[-1], t[0]))
axs[0].set_title('G')
axs[0].set_xlabel(r'$x_R$')
axs[0].set_ylabel(r'$t$')
axs[0].axis('tight')
axs[0].set_ylim(1.5, 0)
axs[1].imshow(G0sub, cmap='gray', vmin=-1e6, vmax=1e6,
    extent=(r[0, 0], r[0, -1], t[-1], t[0]))
axs[1].set_title('G0')
axs[1].set_xlabel(r'$x_R$')
axs[1].set_ylabel(r'$t$')
axs[1].axis('tight')
axs[1].set_ylim(1.5, 0)
```




Out:

```
R_fft: dask.array<from-zarr, shape=(500, 101, 101), dtype=complex64, chunksize=(125,
\hookrightarrow101, 101), chunktype=numpy.ndarray>
(1.5, 0.0)
```

Let's now create an object of the pylops_distributed.waveeqprocessing. Marchenko class and apply redatuming for a single subsurface point vs.

```
# direct arrival window
trav = np.sqrt((vs[0]-r[0])**2+(vs[1]-r[1])**2)/vel
MarchenkoWM = Marchenko(R_fft, nt=nt, dt=dt, dr=dr, wav=wav,
    toff=toff, nsmooth=nsmooth)
f1_inv_minus, f1_inv__plus, p0_minus, g_inv_minus, g_inv_plus = \
    MarchenkoWM.apply_onepoint(trav, G0=G0sub.T, rtm=True, greens=True,
                            dottest=False, **dict(niter=niter, compute=True))
g_inv_tot = g_inv_minus + g_inv_plus
```

We can now compare the result of Marchenko redatuming via LSQR with standard redatuming

```
fig, axs = plt.subplots(1, 3, sharey=True, figsize=(16, 9))
axs[0].imshow(p0_minus.T, cmap='gray', vmin=-5e5, vmax=5e5,
    extent=(r[0, 0], r[0, -1], t[-1], -t[-1]))
axs[0].set_title(r'$p_0^-$')
axs[0].set_xlabel(r'$x_R$')
axs[0].set_ylabel(r'$t$')
axs[0].axis('tight')
axs[0].set_ylim(1.2, 0)
axs[1].imshow(g_inv_minus.T, cmap='gray', vmin=-5e5, vmax=5e5,
    extent=(r[0, 0], r[0, -1], t[-1], -t[-1]))
axs[1].set_title(r'$g^-$')
axs[1].set_xlabel(r'$x_R$')
axs[1].set_ylabel(r'$t$')
axs[1].axis('tight')
axs[1].set_ylim(1.2, 0)
axs[2].imshow(g_inv_plus.T, cmap='gray', vmin=-5e5, vmax=5e5,
    extent=(r[0, 0], r[0, -1], t[-1], -t[-1]))
axs[2].set_title(r'$g^+$')
axs[2].set_xlabel(r'$x_R$')
axs[2].set_ylabel(r'$t$')
axs[2].axis('tight')
axs[2].set_ylim(1.2,0)
fig = plt.figure(figsize=(15, 9))
ax1 = plt.subplot2grid((1, 5), (0, 0), colspan=2)
ax2 = plt.subplot2grid((1, 5), (0, 2), colspan=2)
ax3 = plt.subplot2grid((1, 5), (0, 4))
ax1.imshow(Gsub, cmap='gray', vmin=-5e5, vmax=5e5,
    extent=(r[0, 0], r[0, -1], t[-1], t[0]))
ax1.set_title(r'$G_{true}$')
axs[0].set_xlabel(r'$x_R$')
axs[0].set_ylabel(r'$t$')
ax1.axis('tight')
ax1.set_ylim(1.2, 0)
ax2.imshow(g_inv_tot.T, cmap='gray', vmin=-5e5, vmax=5e5,
```

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```
    extent=(r[0, 0], r[0, -1], t[-1], -t[-1]))
ax2.set_title(r'$G_{est}$')
axs[1].set_xlabel(r'$x_R$')
axs[1].set_ylabel(r'$t$')
ax2.axis('tight')
ax2.set_ylim(1.2, 0)
ax3.plot(Gsub[:, nr//2]/Gsub.max(), t, 'r', lw=5)
ax3.plot(g_inv_tot[nr//2, nt-1:]/g_inv_tot.max(), t, 'k', lw=3)
ax3.set_ylim(1.2, 0)
```




Out:

```
(1.2, 0.0)
```

Total running time of the script: ( 0 minutes 6.286 seconds)

### 1.3 PyLops-distributed API

### 1.3.1 Linear operators

## Basic operators

| MatrixMult(A[, dims, compute, todask, dtype]) | Matrix multiplication. |
| :--- | :--- |
| Identity(N[, M, inplace, compute, todask, dtype]) | Identity operator. |
| Diagonal(diag[, dims, dir, compute, todask, ...]) | Diagonal operator. |
| Transpose(dims, axes[, compute, todask, dtype]) | Transpose operator. |
| Roll(N[, dims, dir, shift, compute, todask, ...]) | Roll along an axis. |
| Restriction(M, iava[, dims, dir, inplace, ...]) | Restriction (or sampling) operator. |
| Spread(dims, dimsd[, table, dtable, ...]) | Spread operator. |
| VStack(ops[, chunks, compute, todask, ...]) | Vertical stacking. |
| HStack(ops[, chunks, compute, todask, dtype]) | Horizontal stacking. |
| BlockDiag(ops[, chunks, compute, todask, dtype]) | Block-diagonal operator. |

## pylops_distributed.MatrixMult

class pylops_distributed.MatrixMult (A, dims=None, compute=(False, False), todask=(False, False), dtype= 'float64')
Matrix multiplication.
Simple wrapper to dask. array. dot for an input matrix A.

## Parameters

A [dask.array.ndarray] Matrix.
dims [tuple, optional] Number of samples for each other dimension of model (model/data will be reshaped and A applied multiple times to each column of the model/data).
compute [tuple, optional] Compute the outcome of forward and adjoint or simply define the graph and return a dask. array
todask [tuple, optional] Apply dask.array.from_array to model and data before applying forward and adjoint respectively
dtype [str, optional] Type of elements in input array.

## Notes

Refer to pylops.basicoperators.MatrixMult for implementation details.

## Attributes

shape [tuple] Operator shape
explicit [b○○l] Operator contains a matrix that can be solved explicitly (True) or not (False)

Methods

| __init__(A[, dims, compute, todask, dtype]) | Initialize this LinearOperator. |
| :---: | :---: |
| adjoint() | Hermitian adjoint. |
| apply_columns(cols) | Apply subset of columns of operator |
| cond([uselobpcg]) | Condition number of linear operator. |
| conj() | Complex conjugate operator |
| div(y[, niter]) | Solve the linear problem $\mathbf{y}=\mathbf{A x}$. |
| div1(y[, niter]) | Solve the linear problem $\mathbf{y}=\mathbf{A x}$. |
| $\operatorname{dot}(\mathrm{x})$ | Matrix-vector multiplication. |
| eigs([neigs, symmetric, niter, uselobpcg]) | Most significant eigenvalues of linear operator. |
| inv() | Return the inverse of $\mathbf{A}$. |
| matmat(X) | Matrix-matrix multiplication. |
| matvec(x) | Matrix-vector multiplication. |
| rmatmat(X) | Adjoint matrix-matrix multiplication. |
| rmatvec(x) | Adjoint Matrix-vector multiplication. |
| todense() | Return dense matrix. |
| tosparse() | Return sparse matrix. |
| transpose() | Transpose this linear operator. |

inv()
Return the inverse of $\mathbf{A}$.

## Returns

Ainv [numpy.ndarray] Inverse matrix.

## pylops_distributed.Identity

class pylops_distributed.Identity ( $N, M=$ None, inplace $=$ True, compute $=($ False, False $)$, todask $=($ False, False $)$, dtype $=$ 'float64')
Identity operator.
Simply move model to data in forward model and viceversa in adjoint mode if $M=N$. If $M>N$ removes last $M-N$ elements from model in forward and pads with 0 in adjoint. If $N>M$ removes last $N-M$ elements from data in adjoint and pads with 0 in forward.

## Parameters

$\mathbf{N}$ [int] Number of samples in data (and model, if $M$ is not provided).
$\mathbf{M}$ [int, optional] Number of samples in model.
inplace [bool, optional] Work inplace (True) or make a new copy (False). By default, data is a reference to the model (in forward) and model is a reference to the data (in adjoint).
compute [tuple, optional] Compute the outcome of forward and adjoint or simply define the graph and return a dask. array
todask [tuple, optional] Apply dask.array.from_array to model and data before applying forward and adjoint respectively
dtype [str, optional] Type of elements in input array.

## Raises

ValueError If M is different from N

## Notes

Refer to pylops.basicoperators.Identity for implementation details.

## Attributes

shape [tuple] Operator shape
explicit [bo○l] Operator contains a matrix that can be solved explicitly (True) or not (False)

## Methods

| __init__(N[, M, inplace, compute, todask dtype]) | Initialize this LinearOperator. |
| :---: | :---: |
| adjoint() | Hermitian adjoint. |
| apply_columns(cols) | Apply subset of columns of operator |
| cond([uselobpcg]) | Condition number of linear operator. |
| conj() | Complex conjugate operator |
| $\operatorname{div}(\mathrm{y}$ [, niter]) | Solve the linear problem $\mathbf{y}=\mathbf{A x}$. |
| div1(y[, niter]) | Solve the linear problem $\mathbf{y}=\mathbf{A x}$. |
| $\operatorname{dot}(\mathrm{x})$ | Matrix-vector multiplication. |
| eigs([neigs, symmetric, niter, uselobpcg]) | Most significant eigenvalues of linear operator. |
| matmat(X) | Matrix-matrix multiplication. |
| matvec(x) | Matrix-vector multiplication. |
| rmatmat(X) | Adjoint matrix-matrix multiplication. |
| rmatvec(x) | Adjoint Matrix-vector multiplication. |
| todense() | Return dense matrix. |
| tosparse() | Return sparse matrix. |
| transpose() | Transpose this linear operator. |

## pylops_distributed.Diagonal

class pylops_distributed.Diagonal (diag, dims=None, dir=0, compute=(False, False), todask $=($ False, False $)$, dtype $=$ 'float64')

## Diagonal operator.

Applies element-wise multiplication of the input vector with the vector diag in forward and with its complex conjugate in adjoint mode.
This operator can also broadcast; in this case the input vector is reshaped into its dimensions dims and the element-wise multiplication with diag is perfomed on the direction dir. Note that the vector diag will need to have size equal to dims [dir].

## Parameters

diag [dask.array.ndarray] Vector to be used for element-wise multiplication.
dims [list, optional] Number of samples for each dimension (None if only one dimension is available)
dir [int, optional] Direction along which multiplication is applied.
compute [tuple, optional] Compute the outcome of forward and adjoint or simply define the graph and return a dask. array. array
todask [tuple, optional] Apply dask.array.from_array to model and data before applying forward and adjoint respectively
dtype [str, optional] Type of elements in input array.

## Notes

Refer to pylops.basicoperators.Diagonal for implementation details.

## Attributes

> shape [tuple] Operator shape
> explicit [bool] Operator contains a matrix that can be solved explicitly (True) or not (False)

## Methods

| init__(diag[, dims, dir, compute, todask, ...]) | Initialize this LinearOperator. |
| :---: | :---: |
| adjoint() | Hermitian adjoint. |
| apply_columns(cols) | Apply subset of columns of operator |
| cond([uselobpcg]) | Condition number of linear operator. |
| conj() | Complex conjugate operator |
| div(y[, niter]) | Solve the linear problem $\mathbf{y}=\mathbf{A x}$. |
| div1(y[, niter]) | Solve the linear problem $\mathbf{y}=\mathbf{A x}$. |
| $\operatorname{dot}(\mathrm{x})$ | Matrix-vector multiplication. |
| eigs([neigs, symmetric, niter, uselobpcg]) | Most significant eigenvalues of linear operator. |
| matmat(X) | Matrix-matrix multiplication. |
| matvec(x) | Matrix-vector multiplication. |
| rmatmat(X) | Adjoint matrix-matrix multiplication. |
| rmatvec(x) | Adjoint Matrix-vector multiplication. |
| todense() | Return dense matrix. |
| tosparse() | Return sparse matrix. |
| transpose() | Transpose this linear operator. |

## Examples using pylops_distributed.Diagonal

- sphx_glr_gallery_plot_diagonal.py


## pylops_distributed.Transpose

class pylops_distributed.Transpose (dims, axes, compute=(False, False), todask=(False, False), dtype= 'float64')
Transpose operator.
Transpose axes of a multi-dimensional array. This operator works with flattened input model (or data), which are however multi-dimensional in nature and will be reshaped and treated as such in both forward and adjoint modes.

## Parameters

dims [tuple, optional] Number of samples for each dimension (None if only one dimension is available)
axes [tuple, optional] Direction along which transposition is applied
compute [tuple, optional] Compute the outcome of forward and adjoint or simply define the graph and return a dask.array. array
todask [tuple, optional] Apply dask.array.from_array to model and data before applying forward and adjoint respectively
dtype [str, optional] Type of elements in input array

## Raises

ValueError If axes contains repeated dimensions (or a dimension is missing)

## Notes

Refer to pylops.basicoperators. Transpose for implementation details.

## Attributes

shape [tuple] Operator shape
explicit [bool] Operator contains a matrix that can be solved explicitly (True) or not (False)

## Methods

| init__(dims, axes[, compute, todask, dtype]) | Initialize this LinearOperator. |
| :--- | :--- |
| adjoint () | Hermitian adjoint. |
| apply_columns(cols) | Apply subset of columns of operator |
| cond([uselobpcg]) | Condition number of linear operator. |
| conj() | Complex conjugate operator |
| div(y[, niter]) | Solve the linear problem $\mathbf{y}=\mathbf{A x .}$ |
| div1(y[, niter]) | Solve the linear problem y $=\mathbf{A x .}$ |
| dot $(x)$ | Matrix-vector multiplication. |
| eigs([neigs, symmetric, niter, uselobpcg]) | Most significant eigenvalues of linear operator. |
| matmat $(X)$ | Matrix-matrix multiplication. |
| matvec $(x)$ | Matrix-vector multiplication. |
| rmatmat $(X)$ | Adjoint matrix-matrix multiplication. |
| rmatvec $(x)$ | Adjoint Matrix-vector multiplication. |
| todense( $)$ | Return dense matrix. |
| tosparse () | Return sparse matrix. |
| transpose () | Transpose this linear operator. |

## pylops_distributed.Roll

class pylops_distributed.Roll(N, dims=None, dir=0, shift=1, compute $=($ False, False $)$, todask $=($ False, False), dtype ='float64')
Roll along an axis.
Roll a multi-dimensional array along a specified direction dir for a chosen number of samples (shift).

## Parameters

$\mathbf{N}$ [int] Number of samples in model.
$\operatorname{dims}$ [list, optional] Number of samples for each dimension (None if only one dimension is available)
dir [int, optional] Direction along which rolling is applied.
shift [int, optional] Number of samples by which elements are shifted
compute [tuple, optional] Compute the outcome of forward and adjoint or simply define the graph and return a dask. array
todask [tuple, optional] Apply dask.array.from_array to model and data before applying forward and adjoint respectively
dtype [str, optional] Type of elements in input array.

## Raises

ValueError If $M$ is different from $N$ and chunks is not provided

## Notes

Refer to pylops.basicoperators.Roll for implementation details.

## Attributes

shape [tuple] Operator shape
explicit [bo○l] Operator contains a matrix that can be solved explicitly (True) or not (False)

## Methods

| init__(N[, dims, dir, shift, compute,.. ]) | Initialize this LinearOperator. |
| :--- | :--- |
| adjoint () | Hermitian adjoint. |
| apply_columns(cols) | Apply subset of columns of operator |
| cond([uselobpcg]) | Condition number of linear operator. |
| conj() | Complex conjugate operator |
| div(y[, niter]) | Solve the linear problem $\mathbf{y}=\mathbf{A x .}$ |
| div1(y[, niter]) | Solve the linear problem y $=\mathbf{A x .}$ |
| dot $(x)$ | Matrix-vector multiplication. |
| eigs([neigs, symmetric, niter, uselobpcg]) | Most significant eigenvalues of linear operator. |
| matmat $(X)$ | Matrix-matrix multiplication. |
| matvec $(x)$ | Matrix-vector multiplication. |
| rmatmat $(X)$ | Adjoint matrix-matrix multiplication. |
| rmatvec $(x)$ | Adjoint Matrix-vector multiplication. |
| todense() | Return dense matrix. |
| tosparse() | Return sparse matrix. |
| transpose() | Transpose this linear operator. |

## pylops_distributed.Restriction

class pylops_distributed.Restriction(M, iava, dims=None, dir=0, inplace=True, compute $=($ False, False $), \quad$ todask $=($ False, False $)$, dtype ='float64')
Restriction (or sampling) operator.
Extract subset of values from input vector at locations iava in forward mode and place those values at locations iava in an otherwise zero vector in adjoint mode.

## Parameters

$\mathbf{M}$ [int] Number of samples in model.
iava [list or numpy. ndarray] Integer indices of available samples for data selection.
dims [list] Number of samples for each dimension (None if only one dimension is available)
dir [int, optional] Direction along which restriction is applied.
inplace [bool, optional] Work inplace (True) or make a new copy (False). By default, data is a reference to the model (in forward) and model is a reference to the data (in adjoint).
compute [tuple, optional] Compute the outcome of forward and adjoint or simply define the graph and return a dask. array
todask [tuple, optional] Apply dask.array.from_array to model and data before applying forward and adjoint respectively
dtype [str, optional] Type of elements in input array.

## Notes

Refer to pylops.basicoperators.Restriction for implementation details.

```
Attributes
    shape [tuple] Operator shape
    explicit [bo○l] Operator contains a matrix that can be solved explicitly (True) or not (False)
```


## Methods

| __init__(M, iava[, dims, dir, inplace, ...]) | Initialize this LinearOperator. |
| :--- | :--- |
| adjoint () | Hermitian adjoint. |
| apply_columns(cols) | Apply subset of columns of operator |
| cond([uselobpcg]) | Condition number of linear operator. |
| conj() | Complex conjugate operator |
| div(y[, niter]) | Solve the linear problem $\mathbf{y}=\mathbf{A x .}$ |
| div1(y[, niter]) | Solve the linear problem $\mathbf{y}=\mathbf{A x .}$ |
| $\operatorname{dot}(\mathrm{x})$ | Matrix-vector multiplication. |
| eigs([neigs, symmetric, niter, uselobpcg]) | Most significant eigenvalues of linear operator. |
| matmat $(X)$ | Matrix-matrix multiplication. |
| matvec(x) | Matrix-vector multiplication. |
| rmatmat $(X)$ | Adjoint matrix-matrix multiplication. |
| rmatvec(x) | Adjoint Matrix-vector multiplication. |
| todense() | Return dense matrix. |
| tosparse() | Return sparse matrix. |
| transpose() | Transpose this linear operator. |

## Examples using pylops_distributed.Restriction

- sphx_glr_gallery_plot_restriction.py


## pylops_distributed.Spread

class pylops_distributed.Spread(dims, dimsd, table=None, dtable=None, compute $=($ False, False), todask=(False, False), dtype ='float64')
Spread operator.

Spread values from the input model vector arranged as a 2-dimensional array of size $\left[n_{x 0} \times n_{t 0}\right]$ into the data vector of size $\left[n_{x} \times n_{t}\right]$. Spreading is performed along parametric curves provided as look-up table of precomputed indices (table) or computed on-the-fly using a function handle (fh).

In adjont mode, values from the data vector are instead stacked along the same parametric curves.

## Parameters

dims [tuple] Dimensions of model vector (vector will be reshaped internally into a twodimensional array of size $\left[n_{x 0} \times n_{t 0}\right.$ ], where the first dimension is the spreading/stacking direction)
dimsd [tuple] Dimensions of model vector (vector will be reshaped internal into a twodimensional array of size $\left[n_{x} \times n_{t}\right]$ )
table [np.ndarray or dask.array.core.Array, optional] Look-up table of indeces of size $\left[n_{x} \times n_{x 0} \times n_{t 0}\right.$ ]
dtable [np.ndarray or dask.array.core.Array, optional] Look-up table of decimals remainders for linear interpolation of same size as dtable
fh [np.ndarray, optional] Function handle that returns an index (and a fractional value in case of interp=True) to be used for spreading/stacking given indices in $x 0$ and $t$ axes (if None use look-up table table)
interp [bool, optional] Apply linear interpolation (True) or nearest interpolation (False) during stacking/spreading along parametric curve. To be used only if engine=' numba', inferred directly from the number of outputs of fh for engine=' numpy '
compute [tuple, optional] Compute the outcome of forward and adjoint or simply define the graph and return a dask. array
todask [tuple, optional] Apply dask.array.from_array to model and data before applying forward and adjoint respectively
dtype [str, optional] Type of elements in input array.

## Raises

KeyError If engine is neither numpy nor numba
NotImplementedError If both table and $f \mathrm{~h}$ are not provided
ValueError If table has shape different from $\left[n_{x 0} \times n_{t} 0 \times n_{x}\right]$

## Notes

Refer to pylops.basicoperators.Spread for implementation details.

## Attributes

shape [tuple] Operator shape
explicit [bool] Operator contains a matrix that can be solved explicitly (True) or not (False)

## Methods

| __init__(dims, dimsd[, table, dtable,,$\ldots])$ | Initialize this LinearOperator. |
| :--- | :--- |
| adjoint () | Hermitian adjoint. |
| apply_columns(cols) | Apply subset of columns of operator |
|  | Continued on next page |

Table 8 - continued from previous page

| cond([uselobpcg]) | Condition number of linear operator. |
| :---: | :---: |
| conj() | Complex conjugate operator |
| div(y[, niter]) | Solve the linear problem $\mathbf{y}=\mathbf{A x}$. |
| div1(y[, niter]) | Solve the linear problem $\mathbf{y}=\mathbf{A x}$. |
| $\operatorname{dot}(\mathrm{x})$ | Matrix-vector multiplication. |
| eigs([neigs, symmetric, niter, uselobpcg]) | Most significant eigenvalues of linear operator. |
| matmat(X) | Matrix-matrix multiplication. |
| matvec(x) | Matrix-vector multiplication. |
| rmatmat(X) | Adjoint matrix-matrix multiplication. |
| rmatvec(x) | Adjoint Matrix-vector multiplication. |
| todense() | Return dense matrix. |
| tosparse() | Return sparse matrix. |
| transpose() | Transpose this linear operator. |

## pylops_distributed.VStack

class pylops_distributed.VStack (ops, chunks=None, compute=(False, False), todask=(False, False), usedelayed $=$ False, dtype $=$ None )
Vertical stacking.
Stack a set of N linear operators vertically.

## Parameters

ops [list] Linear operators to be stacked. Operators must be of pylops_distributed. LinearOperator type for usedelayed=False and pylops. LinearOperator for usedelayed=True
chunks [tuple, optional] Chunks for model and data (an array with a single chunk is created if chunks is not provided)
compute [tuple, optional] Compute the outcome of forward and adjoint or simply define the graph and return a dask. array
todask [tuple, optional] Apply dask.array.from_array to model and data before applying forward and adjoint respectively
usedelayed [bool, optional] Use dask. delayed to parallelize over the N operators. Note that when this is enabled the input model and data should be passed as numpy. ndarray
dtype [str, optional] Type of elements in input array.

## Notes

Refer to pylops.basicoperators.VStack for implementation details.

## Attributes

shape [tuple] Operator shape
explicit [b○○l] Operator contains a matrix that can be solved explicitly (True) or not (False)

## Methods

| init__(ops[, chunks, compute, todask,...$])$ | Initialize this LinearOperator. |
| :--- | :--- |
| adjoint () | Hermitian adjoint. |
| apply_columns(cols) | Apply subset of columns of operator |
| cond([uselobpcg]) | Condition number of linear operator. |
| conj() | Complex conjugate operator |
| div(y[, niter]) | Solve the linear problem $\mathbf{y}=\mathbf{A x .}$ |
| div1(y[, niter]) | Solve the linear problem y $=\mathbf{A x .}$ |
| dot $(x)$ | Matrix-vector multiplication. |
| eigs([neigs, symmetric, niter, uselobpcg]) | Most significant eigenvalues of linear operator. |
| matmat $(X)$ | Matrix-matrix multiplication. |
| matvec $(x)$ | Matrix-vector multiplication. |
| rmatmat $(X)$ | Adjoint matrix-matrix multiplication. |
| rmatvec $(x)$ | Adjoint Matrix-vector multiplication. |
| todense() | Return dense matrix. |
| tosparse() | Return sparse matrix. |
| transpose() | Transpose this linear operator. |

## pylops_distributed.HStack

class pylops_distributed.HStack (ops, chunks=None, compute=(False, False), todask=(False, False), dtype=None)
Horizontal stacking.
Stack a set of N linear operators horizontally.

## Parameters

ops [list] Linear operators to be stacked
chunks [tuple, optional] Chunks for model and data (an array with a single chunk is created if chunks is not provided)
compute [tuple, optional] Compute the outcome of forward and adjoint or simply define the graph and return a dask. array
todask [tuple, optional] Apply dask.array.from_array to model and data before applying forward and adjoint respectively
dtype [str, optional] Type of elements in input array.

## Notes

Refer to pylops.basicoperators. HStack for implementation details.

## Attributes

shape [tuple] Operator shape
explicit [bo○l] Operator contains a matrix that can be solved explicitly (True) or not (False)

## Methods

| __init__(ops[, chunks, compute, todask, dtype]) | Initialize this LinearOperator. |
| :--- | :--- |
| adjoint () | Hermitian adjoint. |

Table 10 - continued from previous page

| apply_columns(cols) | Apply subset of columns of operator |
| :--- | :--- |
| cond([uselobpcg]) | Condition number of linear operator. |
| $\operatorname{conj}()$ | Complex conjugate operator |
| $\operatorname{div}(\mathrm{y}[$ niter] $)$ | Solve the linear problem $\mathbf{y}=\mathbf{A x}$. |
| $\operatorname{div1}(\mathrm{y}[$, niter] $)$ | Solve the linear problem $\mathbf{y}=\mathbf{A x}$. |
| $\operatorname{dot}(\mathrm{x})$ | Matrix-vector multiplication. |
| eigs([neigs, symmetric, niter, uselobpcg $])$ | Most significant eigenvalues of linear operator. |
| matmat $(\mathrm{X})$ | Matrix-matrix multiplication. |
| matvec(x) | Matrix-vector multiplication. |
| rmatmat $(\mathrm{X})$ | Adjoint matrix-matrix multiplication. |
| rmatvec(x) | Adjoint Matrix-vector multiplication. |
| todense() | Return dense matrix. |
| tosparse() | Return sparse matrix. |
| transpose() | Transpose this linear operator. |

## pylops_distributed.BlockDiag

class pylops_distributed.BlockDiag (ops, chunks=None, compute=(False, False), todask $=($ False, False $)$, dtype=None $)$
Block-diagonal operator.
Create a block-diagonal operator from N linear operators.

## Parameters

ops [list] Linear operators to be stacked
chunks [tuple, optional] Chunks for model and data (an array with a single chunk is created if chunks is not provided)
compute [tuple, optional] Compute the outcome of forward and adjoint or simply define the graph and return a dask. array
todask [tuple, optional] Apply dask.array.from_array to model and data before applying forward and adjoint respectively
dtype [str, optional] Type of elements in input array.

## Notes

Refer to pylops.basicoperators.VStack for implementation details.

## Attributes

shape [tuple] Operator shape
explicit [bool] Operator contains a matrix that can be solved explicitly (True) or not (False)

## Methods

| __init__(ops[, chunks, compute, todask, dtype]) | Initialize this LinearOperator. |
| :--- | :--- |
| adjoint() | Hermitian adjoint. |
| apply_columns(cols) | Apply subset of columns of operator |
| cond([uselobpcg]) | Condition number of linear operator. |

Continued on next page

Table 11 - continued from previous page

| conj() | Complex conjugate operator |
| :---: | :---: |
| div(y[, niter]) | Solve the linear problem $\mathbf{y}=\mathbf{A x}$. |
| div1(y[, niter]) | Solve the linear problem $\mathbf{y}=\mathbf{A x}$. |
| $\operatorname{dot}(\mathrm{x})$ | Matrix-vector multiplication. |
| eigs([neigs, symmetric, niter, uselobpcg]) | Most significant eigenvalues of linear operator. |
| matmat(X) | Matrix-matrix multiplication. |
| matvec(x) | Matrix-vector multiplication. |
| rmatmat(X) | Adjoint matrix-matrix multiplication. |
| rmatvec(x) | Adjoint Matrix-vector multiplication. |
| todense() | Return dense matrix. |
| tosparse() | Return sparse matrix. |
| transpose() | Transpose this linear operator. |

## Smoothing and derivatives

| Smoothing1D(nsmooth, dims[, dir, compute,.. ]) | 1D Smoothing. |
| :--- | :--- |
| FirstDerivative(N[, dims, dir, sampling, ...]) | First derivative. |
| SecondDerivative(N[, dims, dir, sampling, ...]) | Second derivative. |
| Laplacian(dims[, dirs, weights, sampling, ...]) | Laplacian. |

## pylops_distributed.Smoothing1D

pylops_distributed.Smoothing1D (nsmooth, dims, dir=0, compute=(False, False), chunks=(None, None), todask=(False, False), dtype ='float64')
1D Smoothing.
Apply smoothing to model (and data) along a specific direction of a multi-dimensional array depending on the choice of dir.

## Parameters

nsmooth [int] Lenght of smoothing operator (must be odd)
dims [tuple or int] Number of samples for each dimension
dir [int, optional] Direction along which smoothing is applied
compute [tuple, optional] Compute the outcome of forward and adjoint or simply define the graph and return a dask. array. array
chunks [tuple, optional] Chunk size for model and data. If provided it will rechunk the model before applying the forward pass and the data before applying the adjoint pass
todask [tuple, optional] Apply dask.array.from_array to model and data before applying forward and adjoint respectively
dtype [str, optional] Type of elements in input array.

## Notes

Refer to pylops.basicoperators.Smoothing1D for implementation details.

## Attributes

shape [tuple] Operator shape
explicit [bo○l] Operator contains a matrix that can be solved explicitly (True) or not (False)

## Examples using pylops_distributed.Smoothing1D

- sphx_glr_gallery_plot_smoothing 1d.py


## pylops_distributed.FirstDerivative

class pylops_distributed.FirstDerivative( $N$, dims=None, dir=0, sampling=1.0, compute $=($ False, False $)$, chunks $=($ None, None $)$, todask=(False, False), dtype='float64')
First derivative.
Apply second-order centered first derivative.

## Parameters

$\mathbf{N}$ [int] Number of samples in model.
dims [tuple, optional] Number of samples for each dimension (None if only one dimension is available)
dir [int, optional] Direction along which smoothing is applied.
sampling [float, optional] Sampling step $d x$.
compute [tuple, optional] Compute the outcome of forward and adjoint or simply define the graph and return a dask.array. array
chunks [tuple, optional] Chunk size for model and data. If provided it will rechunk the model before applying the forward pass and the data before applying the adjoint pass
todask [tuple, optional] Apply dask.array.from_array to model and data before applying forward and adjoint respectively
dtype [str, optional] Type of elements in input array.

## Notes

Refer to pylops.basicoperators.FirstDerivative for implementation details.

## Attributes

shape [tuple] Operator shape
explicit [b००l] Operator contains a matrix that can be solved explicitly (True) or not (False)

## Methods

| __init__(N[, dims, dir, sampling, compute,..$])$ | Initialize this LinearOperator. |
| :--- | :--- |
| adjoint () | Hermitian adjoint. |
| apply_columns(cols) | Apply subset of columns of operator |
| cond([uselobpcg]) | Condition number of linear operator. |
| $\operatorname{conj}()$ | Complex conjugate operator |
| $\operatorname{div}(y[$, niter $])$ | Solve the linear problem $\mathbf{y}=\mathbf{A x}$. |

Continued on next page

Table 13-continued from previous page

| $\operatorname{div1}(\mathrm{y}[$, niter]) | Solve the linear problem $\mathbf{y}=\mathbf{A x}$. |
| :--- | :--- |
| $\operatorname{dot}(\mathrm{x})$ | Matrix-vector multiplication. |
| eigs([neigs, symmetric, niter, uselobpcg]) | Most significant eigenvalues of linear operator. |
| matmat $(\mathrm{X})$ | Matrix-matrix multiplication. |
| matvec $(\mathrm{x})$ | Matrix-vector multiplication. |
| rmatmat $(\mathrm{X})$ | Adjoint matrix-matrix multiplication. |
| rmatvec $(\mathrm{x})$ | Adjoint Matrix-vector multiplication. |
| todense () | Return dense matrix. |
| tosparse () | Return sparse matrix. |
| transpose () | Transpose this linear operator. |

## Examples using pylops_distributed.FirstDerivative

- sphx_glr_gallery_plot_derivative.py


## pylops_distributed.SecondDerivative

class pylops_distributed.SecondDerivative ( $N$, dims $=$ None, dir=0, sampling=1.0, compute $=($ False, False $)$, chunks $=($ None, None $)$, todask=(False, False), dtype = 'float64')
Second derivative.
Apply second-order centered second derivative.

## Parameters

$\mathbf{N}$ [int] Number of samples in model.
dims [tuple, optional] Number of samples for each dimension (None if only one dimension is available)
dir [int, optional] Direction along which smoothing is applied.
sampling [float, optional] Sampling step dx.
compute [tuple, optional] Compute the outcome of forward and adjoint or simply define the graph and return a dask. array. array
chunks [tuple, optional] Chunk size for model and data. If provided it will rechunk the model before applying the forward pass and the data before applying the adjoint pass
todask [tuple, optional] Apply dask.array.from_array to model and data before applying forward and adjoint respectively
dtype [str, optional] Type of elements in input array.

## Notes

Refer to pylops.basicoperators.SecondDerivative for implementation details.

## Attributes

shape [tuple] Operator shape
explicit [bool] Operator contains a matrix that can be solved explicitly (True) or not (False)

## Methods

| init__(N[, dims, dir, sampling, compute,..$])$ | Initialize this LinearOperator. |
| :--- | :--- |
| adjoint () | Hermitian adjoint. |
| apply_columns(cols) | Apply subset of columns of operator |
| cond([uselobpcg]) | Condition number of linear operator. |
| conj() | Complex conjugate operator |
| div(y[, niter]) | Solve the linear problem $\mathbf{y}=\mathbf{A x .}$ |
| div1(y[, niter]) | Solve the linear problem $\mathbf{y}=\mathbf{A x .}$ |
| dot(x) | Matrix-vector multiplication. |
| eigs([neigs, symmetric, niter, uselobpcg]) | Most significant eigenvalues of linear operator. |
| matmat $(X)$ | Matrix-matrix multiplication. |
| matvec(x) | Matrix-vector multiplication. |
| rmatmat $(X)$ | Adjoint matrix-matrix multiplication. |
| rmatvec(x) | Adjoint Matrix-vector multiplication. |
| todense() | Return dense matrix. |
| tosparse() | Return sparse matrix. |
| transpose() | Transpose this linear operator. |

## Examples using pylops_distributed. SecondDerivative

- sphx_glr_gallery_plot_derivative.py


## pylops_distributed.Laplacian

pylops_distributed.Laplacian (dims, $\operatorname{dirs}=(0,1)$, weights $=(1,1), \quad$ sampling=(1, 1), compute $=($ False, False $)$, chunks $=($ None, None $)$, todask $=($ False, False $)$, dtype = 'float64')
Laplacian.
Apply second-order centered Laplacian operator to a multi-dimensional array (at least 2 dimensions are required)

## Parameters

dims [tuple] Number of samples for each dimension.
dirs [tuple, optional] Directions along which laplacian is applied.
weights [tuple, optional] Weight to apply to each direction (real laplacian operator if weights=[1,1])
sampling [tuple, optional] Sampling steps for each direction
compute [tuple, optional] Compute the outcome of forward and adjoint or simply define the graph and return a dask. array. array
chunks [tuple, optional] Chunk size for model and data. If provided it will rechunk the model before applying the forward pass and the data before applying the adjoint pass
todask [tuple, optional] Apply dask.array.from_array to model and data before applying forward and adjoint respectively
dtype [str, optional] Type of elements in input array.

## Returns

120p [pylops.LinearOperator] Laplacian linear operator

## Notes

Refer to pylops.basicoperators.Laplacian for implementation details.

## Examples using pylops_distributed.Laplacian

- sphx_glr_gallery_plot_derivative.py


## Signal processing

| FFT(dims[, dir, nfft, sampling, real, ...]) | One dimensional Fast-Fourier Transform. |
| :--- | :--- |
| Convolve1D(N, h[, offset, dims, dir, ...]) | 1D convolution operator. |
| Fredholm1(G[, nz, saveGt, compute, chunks, ...]) | Fredholm integral of first kind. |

## pylops_distributed.signalprocessing.FFT

class pylops_distributed.signalprocessing.FFT(dims, dir=0, nfft=None, sampling=1.0, real $=$ False, $\quad$ fftshift $=$ False, compute $=($ False, False $), \quad$ chunks $=($ None, None), todask=(None, None), dtype = 'float64')
One dimensional Fast-Fourier Transform.
Apply Fast-Fourier Transform (FFT) along a specific direction dir of a multi-dimensional array of size dim.
Note that the FFT operator is an overload to the dask dask.array.fft.fft (ordask.array.fft.rfft for real models) in forward mode and to the dask dask. array.fft.ifft (or dask.array.fft.irfft for real models) in adjoint mode.
Scaling is properly taken into account to guarantee that the operator is passing the dot-test.


#### Abstract

Note: For a real valued input signal, it is possible to store the values of the Fourier transform at positive frequencies only as values at negative frequencies are simply their complex conjugates. However as the operation of removing the negative part of the frequency axis in forward mode and adding the complex conjugates in adjoint mode is nonlinear, the Linear Operator FTT with real=True is not expected to pass the dot-test. It is thus only advised to use this flag when a forward and adjoint FFT is used in the same chained operator (e.g., FFT. $\mathrm{H} * \mathrm{Op} * F F T$ ) such as in pylops_distributed.waveeqprocessing.mdd.MDC.


## Parameters

dims [tuple] Number of samples for each dimension
dir [int, optional] Direction along which FFT is applied.
nfft [int, optional] Number of samples in Fourier Transform (same as input if $n f f t=$ None)
sampling [float, optional] Sampling step $d t$.
real [bool, optional] Model to which fft is applied has real numbers (True) or not (False). Used to enforce that the output of adjoint of a real model is real.
fftshift [bool, optional] Apply fftshift/ifftshift (True) or not (False)
compute [tuple, optional] Compute the outcome of forward and adjoint or simply define the graph and return a dask. array. array
chunks [tuple, optional] Chunk size for model and data. If provided it will rechunk the model before applying the forward pass and the data before applying the adjoint pass
todask [tuple, optional] Apply dask.array.from_array to model and data before applying forward and adjoint respectively
dtype [str, optional] Type of elements in input array.

## Raises

ValueError If dims is not provided and if dir is bigger than len (dims)

## Notes

Refer to pylops.signalprocessing.FFT for implementation details.

## Attributes

shape [tuple] Operator shape
explicit [bo○l] Operator contains a matrix that can be solved explicitly (True) or not (False)

## Methods

| __init__(dims[, dir, nfft, sampling, real, ...]) | Initialize this LinearOperator. |
| :---: | :---: |
| adjoint() | Hermitian adjoint. |
| apply_columns(cols) | Apply subset of columns of operator |
| cond([uselobpcg]) | Condition number of linear operator. |
| conj() | Complex conjugate operator |
| div(y[, niter]) | Solve the linear problem $\mathbf{y}=\mathbf{A x}$. |
| div1(y[, niter]) | Solve the linear problem $\mathbf{y}=\mathbf{A x}$. |
| dot(x) | Matrix-vector multiplication. |
| eigs([neigs, symmetric, niter, uselobpcg]) | Most significant eigenvalues of linear operator. |
| matmat(X) | Matrix-matrix multiplication. |
| matvec(x) | Matrix-vector multiplication. |
| rmatmat(X) | Adjoint matrix-matrix multiplication. |
| rmatvec(x) | Adjoint Matrix-vector multiplication. |
| todense() | Return dense matrix. |
| tosparse() | Return sparse matrix. |
| transpose() | Transpose this linear operator. |

## pylops_distributed.signalprocessing.Convolve1D

class pylops_distributed.signalprocessing.Convolve1D ( $N, h$, offset=0, dims=None, dir $=0, \quad$ compute $=($ False, False), chunks=(None, None), todask $=($ False, False), dtype = 'float64')
1D convolution operator.
Apply one-dimensional convolution with a compact filter to model (and data) along a specific direction of a
multi-dimensional array depending on the choice of dir.
Note that if a multi-dimensional array is provided the array can also be chuncked along the direction dir where convolution is performed. In this case, dask handles the communication of borders between neighboring blocks.

## Parameters

$\mathbf{N}$ [int] Number of samples in model.
h [numpy . ndarray] 1d compact filter to be convolved to input signal
offset [int] Index of the center of the compact filter
dims [tuple] Number of samples for each dimension (None if only one dimension is available)
dir [int, optional] Direction along which convolution is applied
compute [tuple, optional] Compute the outcome of forward and adjoint or simply define the graph and return a dask. array. array
chunks [tuple, optional] Chunk size for model and data. If provided it will rechunk the model before applying the forward pass and the data before applying the adjoint pass
todask [tuple, optional] Apply dask.array.from_array to model and data before applying forward and adjoint respectively
dtype [str, optional] Type of elements in input array.

## Raises

ValueError If offset is bigger than len (h) - 1

## Notes

Refer to pylops.signalprocessing. Convolve1D for implementation details.

## Attributes

shape [tuple] Operator shape
explicit [bool] Operator contains a matrix that can be solved explicitly (True) or not (False)

## Methods

| __init__(N, h[, offset, dims, dir, compute, ...]) | Initialize this LinearOperator. |
| :---: | :---: |
| adjoint() | Hermitian adjoint. |
| apply_columns(cols) | Apply subset of columns of operator |
| cond([uselobpcg]) | Condition number of linear operator. |
| conj() | Complex conjugate operator |
| div(y[, niter]) | Solve the linear problem $\mathbf{y}=\mathbf{A x}$. |
| div1(y[, niter]) | Solve the linear problem $\mathbf{y}=\mathbf{A x}$. |
| $\operatorname{dot}(\mathrm{x})$ | Matrix-vector multiplication. |
| eigs([neigs, symmetric, niter, uselobpcg]) | Most significant eigenvalues of linear operator. |
| matmat(X) | Matrix-matrix multiplication. |
| matvec(x) | Matrix-vector multiplication. |
| rmatmat(X) | Adjoint matrix-matrix multiplication. |

Continued on next page

Table 17-continued from previous page

| rmatvec(x) | Adjoint Matrix-vector multiplication. |
| :--- | :--- |
| todense() | Return dense matrix. |
| tosparse() | Return sparse matrix. |
| transpose () | Transpose this linear operator. |

## Examples using pylops_distributed.signalprocessing. Convolve1D

- sphx_glr_gallery_plot_smoothing1d.py
- sphx_glr_gallery_plot_convolve.py


## pylops_distributed.signalprocessing.Fredholm1

class pylops_distributed.signalprocessing.Fredholm1 ( $G, \quad n z=1, \quad$ saveGt=True,

| compute $=($ False, | False $),$ |
| :--- | :--- |
| chunks $=($ None, | None, |
| todas $=($ None, | None $),$ | dtype = 'float64')

Fredholm integral of first kind.
Implement a multi-dimensional Fredholm integral of first kind. Note that if the integral is two dimensional, this can be directly implemented using pylops.basicoperators.MatrixMult. A multi-dimensional Fredholm integral can be performed as a pylops.basicoperators.BlockDiag operator of a series of pylops.basicoperators.MatrixMult. However, here we take advantage of the structure of the kernel and perform it in a more efficient manner.

## Parameters

$\mathbf{G}$ [numpy. ndarray] Multi-dimensional convolution kernel of size $\left[n_{\text {slice }} \times n_{x} \times n_{y}\right.$ ]
$\mathbf{n z}$ [numpy.ndarray, optional] Additional dimension of model
saveGt [bool, optional] Save $G$ and $G^{\wedge} H$ to speed up the computation of adjoint (True) or create $G^{\wedge} H$ on-the-fly (False) Note that saveGt=True will double the amount of required memory
compute [tuple, optional] Compute the outcome of forward and adjoint or simply define the graph and return a dask. array
chunks [tuple, optional] Chunk size for model and data. If provided it will rechunk the model before applying the forward pass and the data before applying the adjoint pass
todask [tuple, optional] Apply dask.array.from_array to model and data before applying forward and adjoint respectively
dtype [str, optional] Type of elements in input array.

## Notes

Refer to pylops.signalprocessing.Identity for implementation details.

## Attributes

shape [tuple] Operator shape
explicit [ $\mathrm{b} \circ \circ \mathrm{l}$ ] Operator contains a matrix that can be solved explicitly (True) or not (False)

## Methods

| init__(G[, nz, saveGt, compute, chunks, $\ldots$ ] $)$ | Initialize this LinearOperator. |
| :--- | :--- |
| adjoint () | Hermitian adjoint. |
| apply_columns(cols) | Apply subset of columns of operator |
| cond([uselobpcg]) | Condition number of linear operator. |
| conj() | Complex conjugate operator |
| div(y[, niter]) | Solve the linear problem $\mathbf{y}=\mathbf{A x .}$ |
| div1(y[, niter]) | Solve the linear problem y $=\mathbf{A x .}$ |
| dot(x) | Matrix-vector multiplication. |
| eigs([neigs, symmetric, niter, uselobpcg]) | Most significant eigenvalues of linear operator. |
| matmat $(X)$ | Matrix-matrix multiplication. |
| matvec $(x)$ | Matrix-vector multiplication. |
| rmatmat $(X)$ | Adjoint matrix-matrix multiplication. |
| rmatvec $(x)$ | Adjoint Matrix-vector multiplication. |
| todense() | Return dense matrix. |
| tosparse() | Return sparse matrix. |
| transpose() | Transpose this linear operator. |

## Wave-Equation processing

| MDC(G, nt, nv[, dt, dr, twosided, saveGt, ...]) | Multi-dimensional convolution. |
| :--- | :--- |
| Marchenko(R, nt[, dt, dr, wav, toff, ...]) | Marchenko redatuming |
| Demigration(z, $\mathrm{x}, \mathrm{t}$, srcs, recs, vel, wav, ...) | Demigration operator. |

## pylops_distributed.waveeqprocessing.MDC

pylops_distributed.waveeqprocessing.MDC(G, $n t, \quad n v, \quad d t=1.0, \quad d r=1.0, \quad t w o s i d e d=T r u e$, saveGt=True, conj=False, prescaled=False, compute $=($ False, False $)$, todask $=($ False, False $)$ )
Multi-dimensional convolution.
Apply multi-dimensional convolution between two datasets. Model and data should be provided after flattening 2- or 3-dimensional arrays of size $\left[n_{t} \times n_{r}\left(\times n_{v s}\right)\right]$ and $\left[n_{t} \times n_{s}\left(\times n_{v s}\right)\right]$ (or $2 * n_{t}-1$ for twosided=True), respectively.

## Parameters

G [dask.array.ndarray] Multi-dimensional convolution kernel in frequency domain of size $\left[n_{f m a x} \times n_{s} \times n_{r}\right]$
nt [int] Number of samples along time axis
nv [int] Number of samples along virtual source axis
dt [float, optional] Sampling of time integration axis
dr [float, optional] Sampling of receiver integration axis
twosided [bool, optional] MDC operator has both negative and positive time (True) or only positive (False)
saveGt [bool, optional] Save $G$ and $G^{\wedge} H$ to speed up the computation of adjoint of pylops_distributed.signalprocessing. FredholmI (True) or create G^H
on-the-fly (False) Note that saveGt=True will be faster but double the amount of required memory
conj [str, optional] Perform Fredholm integral computation with complex conjugate of $G$
prescaled [bo○l, optional] Apply scaling to kernel (False) or not (False) when performing spatial and temporal summations. In case prescaled=True, the kernel is assumed to have been pre-scaled when passed to the MDC routine.
compute [tuple, optional] Compute the outcome of forward and adjoint or simply define the graph and return a dask. array
todask [tuple, optional] Apply dask.array.from_array to model and data before applying forward and adjoint respectively

## Notes

Refer to pylops.waveeqprocessing.MDC for implementation details.

## pylops_distributed.waveeqprocessing.Marchenko

class pylops_distributed.waveeqprocessing.Marchenko ( $R, \quad n t, \quad d t=0.004, \quad d r=1.0$, wav=None, $\quad$ toff $=0.0, \quad n s$ mooth=10, saveRt=True, prescaled=False, dtype='float 32 ')
Marchenko redatuming
Solve multi-dimensional Marchenko redatuming problem using scipy.sparse.linalg.lsqr iterative solver.

## Parameters

$\mathbf{R}$ [dask.array] Multi-dimensional reflection response in frequency domain of size $\left[n_{f \max } \times n_{s} \times n_{r}\right]$. Note that the reflection response should have already been multiplied by 2 .
nt [float, optional] Number of samples in time
dt [float, optional] Sampling of time integration axis
dr [float, optional] Sampling of receiver integration axis
wav [numpy.ndarray, optional] Wavelet to apply to direct arrival when created using trav
toff [float, optional] Time-offset to apply to traveltime
nsmooth [int, optional] Number of samples of smoothing operator to apply to window
saveRt [bool, optional] Save $R$ and $R^{\wedge} H$ to speed up the computation of the adjoint of pylops_distributed.signalprocessing. Fredholml (True) or create R^H on-the-fly (False) Note that saveRt=True will be faster but double the amount of required memory
prescaled [bool, optional] Apply scaling to R (False) or not (False) when performing spatial and temporal summations within the pylops.waveeqprocessing.MDC operator. In case prescaled=True, the $R$ is assumed to have been pre-scaled by the user.
dtype [bo○l, optional] Type of elements in input array.

## Raises

TypeError If $t$ is not numpy. ndarray.

## See also:

MDC Multi-dimensional convolution

## Notes

Refer to pylops.waveeqprocessing. Marchenko for implementation details.

## Attributes

ns [int] Number of samples along source axis
nr [int] Number of samples along receiver axis
shape [tuple] Operator shape
explicit [bool] Operator contains a matrix that can be solved explicitly (True) or not (False)

## Methods



## Parameters

trav [numpy. ndarray] Traveltime of first arrival from subsurface point to surface receivers of size $\left[n_{r} \times 1\right]$
dist: :obj:‘numpy.ndarray', optional Distance between subsurface point to surface receivers of size $[n r \times 1]$ (if provided the analytical direct arrival will be computed using a 3d formulation)

G0 [numpy . ndarray, optional] Direct arrival in time domain of size $\left[n_{r} \times n_{t}\right.$ ] (if None, create arrival using trav)
nfft [int, optional] Number of samples in fft when creating the analytical direct wave
rtm [bool, optional] Compute and return rtm redatuming
greens [bo○l, optional] Compute and return Green's functions
dottest [bo○l, optional] Apply dot-test
**kwargs_cgls Arbitrary keyword arguments for pylops_distributed. optimization.cg.cgls solver

## Returns

f1_inv_minus [dask.array] Inverted upgoing focusing function of size $\left[n_{r} \times n_{t}\right.$ ]
f1_inv_plus [dask.array] Inverted downgoing focusing function of size $\left[n_{r} \times n_{t}\right.$ ]
p0_minus [dask.array] Single-scattering standard redatuming upgoing Green's function of size $\left[n_{r} \times n_{t}\right]$
g_inv_minus [dask.array] Inverted upgoing Green's function of size $\left[n_{r} \times n_{t}\right.$ ]
g_inv_plus [dask.array] Inverted downgoing Green's function of size $\left[n_{r} \times n_{t}\right.$ ]
apply_multiplepoints (trav, dist=None, G0=None, nfft=None, rtm=False, greens=False, dottest=False, **kwargs_cgls)
Marchenko redatuming for multiple points
Solve the Marchenko redatuming inverse problem for multiple points given their direct arrival traveltime curves (trav) and waveforms (G0).

## Parameters

trav [numpy. ndarray] Traveltime of first arrival from subsurface points to surface receivers of size $\left[n_{r} \times n_{v s}\right]$
dist: :obj:‘numpy.ndarray‘, optional Distance between subsurface point to surface receivers of size $\left[n_{r} \times n_{v s}\right]$ (if provided the analytical direct arrival will be computed using a 3d formulation)

G0 [numpy . ndarray, optional] Direct arrival in time domain of size $\left[n_{r} \times n_{v s} \times n_{t}\right.$ ] (if None, create arrival using trav)
nfft [int, optional] Number of samples in fft when creating the analytical direct wave
rtm [bool, optional] Compute and return rtm redatuming
greens [bool, optional] Compute and return Green's functions
dottest [b○○l, optional] Apply dot-test
**kwargs_cgls Arbitrary keyword arguments for pylops_distributed. optimization.cg.cgls solver

## Returns

f1_inv_minus [numpy . ndarray] Inverted upgoing focusing function of size $\left[n_{r} \times n_{v s} \times\right.$ $\left.n_{t}\right]$
f1_inv_plus [numpy . ndarray] Inverted downgoing focusing functionof size $\left[n_{r} \times n_{v s} \times\right.$
$n_{t}$ ] $n_{t}$ ]
p0_minus [numpy.ndarray] Single-scattering standard redatuming upgoing Green's function of size $\left[n_{r} \times n_{v s} \times n_{t}\right]$
g_inv_minus [numpy . ndarray] Inverted upgoing Green's function of size $\left[n_{r} \times n_{v s} \times n_{t}\right.$ ]
g_inv_plus [numpy.ndarray] Inverted downgoing Green's function of size $\left[n_{r} \times n_{v s} \times\right.$ $\left.n_{t}\right]$

## Examples using pylops_distributed.waveeqprocessing.Marchenko

- Marchenko redatuming by inversion


## pylops_distributed.waveeqprocessing.Demigration

pylops_distributed.waveeqprocessing. Demigration $(z, x, t, \operatorname{srcs}$, recs, vel, wav, wavcenter, $y=$ None, mode ='eikonal', trav=None, nprocesses=None, client=None)
Demigration operator.
Seismic demigration/migration operator.

## Parameters

$\mathbf{z}$ [numpy.ndarray] Depth axis
$\mathbf{x}$ [numpy.ndarray] Spatial axis
t [numpy. ndarray] Time axis for data
srcs [numpy.ndarray] Sources in array of size $\left[2 / 3 \times n_{s}\right.$ ]
recs [numpy. ndarray] Receivers in array of size $\left[2 / 3 \times n_{r}\right.$ ]
vel [numpy. ndarray or float] Velocity model of size $\left[\left(n_{y} \times\right) n_{x} \times n_{z}\right]$ (or constant)
wav [numpy. ndarray] Wavelet
wavcenter [int] Index of wavelet center
$\mathbf{y}$ [numpy.ndarray] Additional spatial axis (for 3-dimensional problems)
mode [str, optional] Computation mode (analytic, eikonal or byot, see Notes for more details)
trav [numpy.ndarray or dask.array.core. Array, optional] Traveltime table of size $\left[n_{r} * n_{s} \times\left(n_{y} *\right) n_{x} * n_{z}\right]$ To be provided only when mode='byot '
nprocesses [str, optional] Number of processes to split computations
client [dask.distributed.client.Client, optional] Dask client. If provided, the traveltime computation will be persisted.

## Returns

demop [pylops.LinearOperator] Demigration/Migration operator

## Raises

NotImplementedError If mode is neither analytic, eikonal, or byot

## Notes

The demigration operator synthetizes seismic data given from a propagation velocity model $v$ and a reflectivity model $m$. In forward mode:

$$
d\left(\mathbf{x}_{\mathbf{r}}, \mathbf{x}_{\mathbf{s}}, t\right)=w(t) * \int_{V} G\left(\mathbf{x}, \mathbf{x}_{\mathbf{s}}, t\right) G\left(\mathbf{x}_{\mathbf{r}}, \mathbf{x}, t\right) m(\mathbf{x}) d \mathbf{x}
$$

where $m(\mathbf{x})$ is the model and it represents the reflectivity at every location in the subsurface, $G\left(\mathbf{x}, \mathbf{x}_{\mathbf{s}}, t\right)$ and $G\left(\mathbf{x}_{\mathbf{r}}, \mathbf{x}, t\right)$ are the Green's functions from source-to-subsurface-to-receiver and finally $w(t)$ is the wavelet. Depending on the choice of mode the Green's function will be computed and applied differently:

- mode=analytic or mode=eikonal: traveltime curves between source to receiver pairs are computed for every subsurface point and Green's functions are implemented from traveltime look-up tables, placing the reflectivity values at corresponding source-to-receiver time in the data.
- byot: bring your own table. Traveltime table provided directly by user using trav input parameter. Green's functions are then implemented in the same way as previous options.
The adjoint of the demigration operator is a migration operator which projects data in the model domain creating an image of the subsurface reflectivity.


### 1.3.2 Solvers

## Low-level solvers

| $c g(\mathrm{~A}, \mathrm{y}[, \mathrm{x}$, niter, tol, compute, client $])$ | Conjugate gradient |
| :--- | :--- |
| $c g l s(\mathrm{~A}, \mathrm{y}[, \mathrm{x}$, niter, damp, tol, compute, $\ldots \mathrm{]})$ | Conjugate gradient least squares |

## pylops_distributed.optimization.cg.cg

pylops_distributed.optimization.cg.cg(A,y, x=None, niter $=10$, tol=le-05, compute $=$ False, client $=$ None)
Conjugate gradient
Solve a system of equations given the square operator $A$ and data $y$ using conjugate gradient iterations.

## Parameters

A [pylops_distributed.LinearOperator] Operator to invert of size $[N \times N]$
$\mathbf{y}$ [dask.array] Data of size $[N \times 1$ ]
x0 [dask.array, optional] Initial guess
niter [int, optional] Number of iterations
tol [float, optional] Tolerance on residual norm
compute [tuple, optional] Compute intermediate results at the end of every iteration
client [dask.distributed.client.Client, optional] Dask client. If provided when compute $=$ None each iteration is persisted. This is the preferred method to avoid repeating computations.

## Returns

$\mathbf{x}$ [dask. array] Estimated model
iter [int] Number of executed iterations

## Notes

Solve the the following problem using conjugate gradient iterations:

$$
\mathbf{y}=\mathbf{A} \mathbf{x}
$$

Note that early stopping based on tol is activated only when client is provided or compute=True. The formed approach is preferred as it avoid repeating computations along the compute tree.

## pylops_distributed.optimization.cg.cgls

pylops_distributed.optimization.cg.cgls (A, y, $x=$ None, niter $=10$, damp $=0.0$, tol=0.0001, compute $=$ False, client $=$ None )
Conjugate gradient least squares
Solve an overdetermined system of equations given an operator $A$ and data $y$ using conjugate gradient iterations.

## Parameters

A [pylops_distributed.LinearOperator] Operator to invert of size $[N \times N]$
$\mathbf{y}$ [dask. array] Data of size $[N \times 1$ ]
x0 [dask.array, optional] Initial guess
niter [int, optional] Number of iterations
damp [float, optional] Damping coefficient
tol [float, optional] Tolerance on residual norm
compute [tuple, optional] Compute intermediate results at the end of every iteration
client [dask.distributed.client. Client, optional] Dask client. If provided when compute=None each iteration is persisted. This is the preferred method to avoid repeating computations.

## Returns

$\mathbf{x}$ [dask. array] Estimated model
iit [int] Number of executed iterations

## Notes

Minimize the following functional using conjugate gradient iterations:

$$
J=\|\mathbf{y}-\mathbf{A} \mathbf{x}\|^{2}+\epsilon\|\mathbf{x}\|^{2}
$$

where $\epsilon$ is the damping coefficient.
Note that early stopping based on tol is activated only when client is provided or compute=True. The formed approach is preferred as it avoid repeating computations along the compute tree.

### 1.3.3 Applications

## Wave-Equation processing

| $\operatorname{LSM}(\mathrm{z}, \mathrm{x}, \mathrm{t}$, srcs, recs, vel, wav, wavcenter) | Least-squares Migration (LSM). |
| :---: | :---: |
| pylops_distributed.waveeqprocessing.LSM |  |
| class pylops_distributed.wavee | g. LSM $(z, x, t$, srcs, recs, vel, wav, wavcenter, $y=$ None, mode='eikonal', trav=None, dottest=False, nprocesses=None, client=None) |

Solve seismic migration as inverse problem given smooth velocity model vel and an acquisition setup identified by sources (src) and receivers (recs)

## Parameters

$\mathbf{z}$ [numpy.ndarray] Depth axis
$\mathbf{x}$ [numpy.ndarray] Spatial axis
$\mathbf{t}$ [numpy. ndarray] Time axis for data
srcs [numpy.ndarray] Sources in array of size $\left[2 / 3 \times n_{s}\right.$ ]
recs [numpy.ndarray] Receivers in array of size $\left[2 / 3 \times n_{r}\right]$
vel [numpy.ndarray or float] Velocity model of size $\left[\left(n_{y} \times\right) n_{x} \times n_{z}\right]$ (or constant)
wav [numpy.ndarray] Wavelet
wavcenter [int] Index of wavelet center
$\mathbf{y}$ [numpy.ndarray] Additional spatial axis (for 3-dimensional problems)
mode [numpy.ndarray, optional] Computation mode (eikonal, analytic - only for constant velocity)
trav [numpy.ndarray, optional] Traveltime table of size $\left[\left(n_{y} *\right) n_{x} * n_{z} \times n_{r} * n_{s}\right]$ (to be provided if mode='byot')
dottest [bo○l, optional] Apply dot-test
enginetrav [str, optional] Engine used for traveltime computation when mode='eikonal' (numpy and dask supported)
engine [str, optional] Engine used for pylops.basicoperators.Spread computation in forward and adjoint modelling operations (numpy, numba, or dask)
nprocesses [str, optional] Number of processes to split computations on (if engine=dask)

## See also:

pylops.waveeqprocessing.Demigration Demigration operator

## Notes

Inverting a demigration operator is generally referred in the literature as least-squares migration (LSM) as historically a least-squares cost function has been used for this purpose. In practice any other cost function could be used, for examples if solver='pylops.optimization.sparsity.FISTA' a sparse representation of reflectivity is produced as result of the inversion.

Finally, it is worth noting that in the first iteration of an iterative scheme aimed at inverting the demigration operator, a projection of the recorded data in the model domain is performed and an approximate (band-limited) image of the subsurface is created. This process is referred to in the literature as migration.

## Attributes

Demop [pylops.LinearOperator] Demigration operator

## Methods

| __init__(z, $x, t$, srcs, recs, vel, wav,..$)$ | Initialize self. |
| :--- | :--- |
| solve $(\mathrm{d}[$, solver $])$ | Solve least-squares migration equations with chosen <br> solver |

solve ( $d$, solver $=<$ function lsqr>, $* * k$ wargs_solver)
Solve least-squares migration equations with chosen solver

## Parameters

d [numpy . ndarray] Input data of size $\left[n_{s} \times n_{r} \times n_{t}\right.$ ]
solver [func, optional] Solver to be used for inversion
**kwargs_solver Arbitrary keyword arguments for chosen solver

## Returns

$\operatorname{minv}$ [np. ndarray] Inverted reflectivity model of size $\left[\left(n_{y} \times\right) n_{x} \times n_{z}\right]$

### 1.4 PyLops-distributed Utilities

Alongside with its Linear Operators and Solvers, PyLops contains also a number of auxiliary routines performing universal tasks that are used by several operators or simply within one or more Tutorials for the preparation of input data and subsequent visualization of results.

### 1.4.1 Shared

## Backends

backend. dask([hardware, client, processes, ...]) Dask backend initialization.
pylops_distributed.utils.backend.dask
pylops_distributed.utils.backend.dask(hardware='single', client=None, processes=False, n_workers $=1$, threads_per_worker $=1, * *$ kwargscluster)
Dask backend initialization.
Create connection to drive computations using Dask distributed.

## Parameters

hardware [str, optional] Hardware used to run Dask distributed. Currently available options are single for single-machine distribution, ssh for SSH-bases multi-machine distribution and p.bs for PBS-bases multi-machine distribution
client [str, optional] Name of scheduler (use None for hardware=single).
processes [str, optional] Whether to use processes (True) or threads (False).
n_workers [int, optional] Number of workers
threads_per_worker [int, optional] Number of threads per each worker
kwargscluster: Additional parameters to be passed to the cluster creation routine

## Returns

client [dask.distributed.client. Client] Client cluster : Cluster

## Raises

NotImplementedError If hardware is not single, ssh, or pbs

## Dot-test

dottest(Op, nr, nc, chunks[, tol, ...]) Dot test.

## pylops_distributed.utils.dottest

pylops_distributed.utils.dottest(Op, nr, nc, chunks, tol=le-06, complexflag=0, raiseer-
Dot test.
Generate random vectors $\mathbf{u}$ and $\mathbf{v}$ and perform dot-test to verify the validity of forward and adjoint operators. This test can help to detect errors in the operator implementation.

## Parameters

Op [pylops.LinearOperator] Linear operator to test.
$\mathbf{n r}$ [int] Number of rows of operator (i.e., elements in data)
nc [int] Number of columns of operator (i.e., elements in model)
chunks [tuple, optional] Chunks for data and model
tol [float, optional] Dottest tolerance
complexflag [bool, optional] generate random vectors with real (0) or complex numbers (1: only model, 2 : only data, 3:both)
raiseerror [ $\mathrm{b} \circ \circ \mathrm{l}$, optional] Raise error or simply return Fal se when dottest fails
verb [bool, optional] Verbosity

## Raises

ValueError If dot-test is not verified within chosen tolerance.

## Notes

A dot-test is mathematical tool used in the development of numerical linear operators.
More specifically, a correct implementation of forward and adjoint for a linear operator should verify the following equality within a numerical tolerance:

$$
(\mathbf{O p} * \mathbf{u})^{H} * \mathbf{v}=\mathbf{u}^{H} *\left(\mathbf{O} \mathbf{p}^{H} * \mathbf{v}\right)
$$

### 1.5 Contributing

Contributions are welcome and greatly appreciated!
Follow the instructions in our main repository

### 1.6 Changelog

### 1.6.1 Version 0.2.0

Released on: 06/06/2020

- Added prescaled input parameter to pylops_distributed.waveeqprocessing.MDC and pylops_distributed.waveeqprocessing. Marchenko
- Added dtype parameter to the FFT calls in the definition of the pylops_distributed. waveeqprocessing.MDD operation. This ensure that the type of the real part of $G$ input is enforced to the output vectors of the forward and adjoint operations.
- Changed handling of dtype in pylops_distributed.signalprocessing.FFT to ensure that the type of the input vector is retained when applying forward and adjoint.
- Added PBS backend to pylops_distributed.utils.backend.dask


### 1.6.2 Version 0.1.0

Released on: 09/02/2020

- Added pylops_distributed. Restriction operator
- Added pylops_distributed.signalprocessing. ConvolvelD and pylops_distributed. signalprocessing.FFT2D operators
- Improved efficiency of pylops_distributed.signalprocessing.Fredholml when saveGt=False
- Adapted pylops_distributed.optimization.cg.cg and pylops_distributed. optimization.cg.cgls solvers for complex numbers


### 1.6.3 Version 0.0.0

Released on: 01/09/2019

- First official release.


### 1.7 Roadmap

Coming soon...

### 1.8 Contributors

- Matteo Ravasi, mrava87


## Index

## A

```
apply_multiplepoints()
    (py- Laplacian() (in module pylops_distributed), 27
    lops_distributed.waveeqprocessing.Marchenko LSM (class in pylops_distributed.waveeqprocessing), 38
    method), }3
apply_onepoint()
    (py-
        lops_distributed.waveeqprocessing.Marchenko
        method),34
```

```
B
BlockDiag(class in pylops_distributed),23
```


## C

```
cg () (in module pylops_distributed.optimization.cg), 37 cgls() (in module pylops_distributed.optimization.cg), 38
Convolve1D (class in py- S
lops_distributed.signalprocessing), 29
```


## $D$

```
dask () (in module pylops_distributed.utils.backend), 40 Demigration() (in module py-
lops_distributed.waveeqprocessing), 36
Diagonal (class in pylops_distributed), 15
dottest () (in module pylops_distributed.utils), 41
```


## F

```
FFT (class in pylops_distributed.signalprocessing), 28
FirstDerivative (class in pylops_distributed), 25
Fredholm1 (class in pylops_distributed.signalprocessing), 31
```


## H

```
HStack (class in pylops_distributed), 22
```


## |

Identity (class in pylops_distributed), 14
inv () (pylops_distributed.MatrixMult method), 14

